
بسم الله الرحمن الرحيم

Benha University
Shoubra Faculty of Engineering

Energy & sustainable energy Dep.
1st year

13.04.2019 – Week 10

Compression test
&
Bending test

Dr. Mahmoud Khedr

Outline

- Compression test.
- Difficulties of the Compression test.
- Behavior of Metals under Compression.
- Stress distribution over the cross section under tension/compression.
- Theory of simple bending.
- The mechanical properties after performing of the bending test.
- Cold Bent Test

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Behavior of Metals under Static Compressive Stresses

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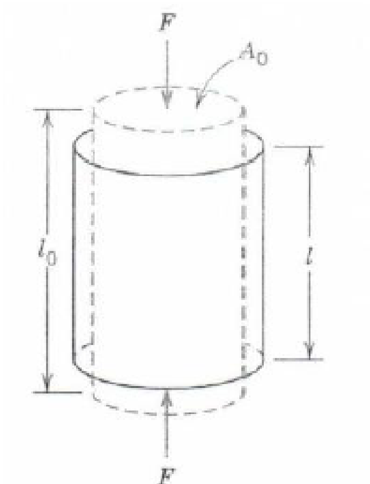
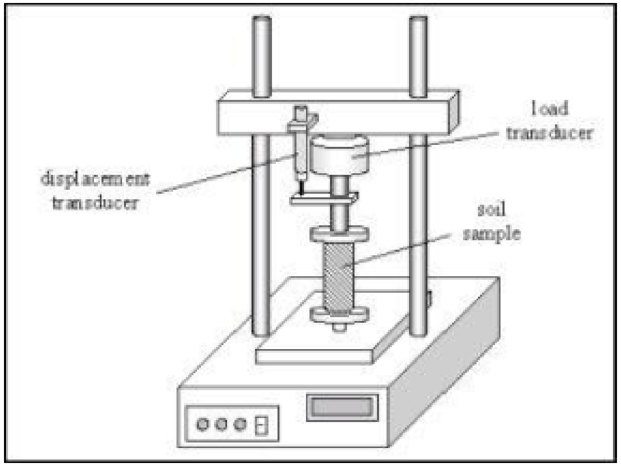
Introduction

•A compression test is conducted in a manner similar to the tensile test, except that the force is compressive and the specimen contracts along the direction of the stress.

• ***Tensile tests are more common*** because they are easier to perform; also, for most materials used in structural applications, very little additional information is obtained from compressive tests.



Schematic illustration of how a compressive load produces contraction and a negative linear strain.



Introduction

The compression test is usually carried out on "non-metal" materials such as concrete, timber, tiles, natural stones to determine compressive strength only.

Rarely it is carried out on metals since most of the important mechanical properties are easily and accurately determined from Tension Test.

Compression test ***may*** be carried out on metals to determine yield stress, Young's modulus, etc.

Compressive tests are used when a material's behavior under large and permanent loading and strains is desired, or material is brittle in tension.

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Watching a compression testing practice

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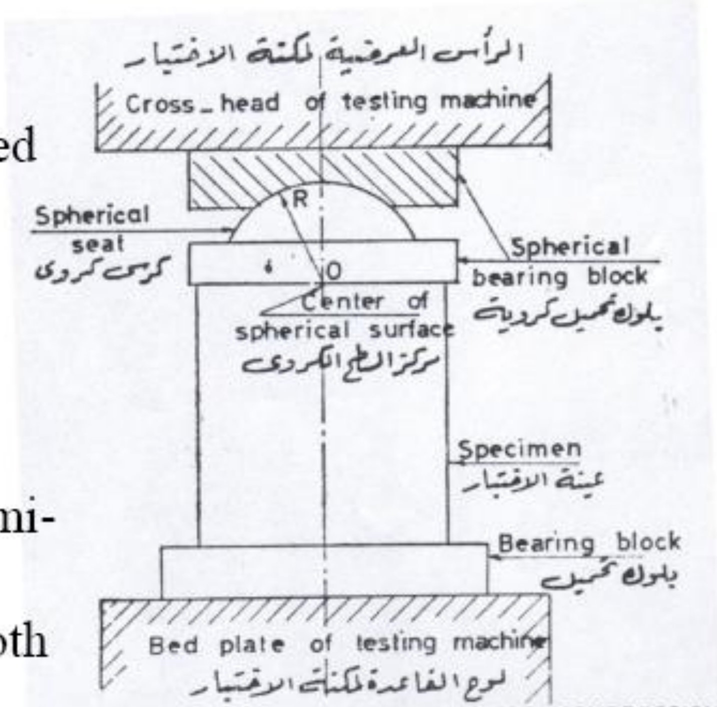
Difficulties of The Compression test

The application of perfectly axial load.

If the load on a specimen is applied through the center of gravity of its cross section, it is called an axial load.

A load at any other point in the cross section is known as an eccentric load.

The testing machine should be equipped with hemi-spherical end to prevent load concentration and apply stresses that are always perpendicular to both ends of the specimen.

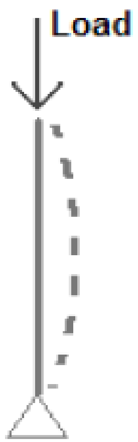


SPHERICAL BEARING BLOCK FOR COMPRESSION TEST

Difficulties of The Compression test

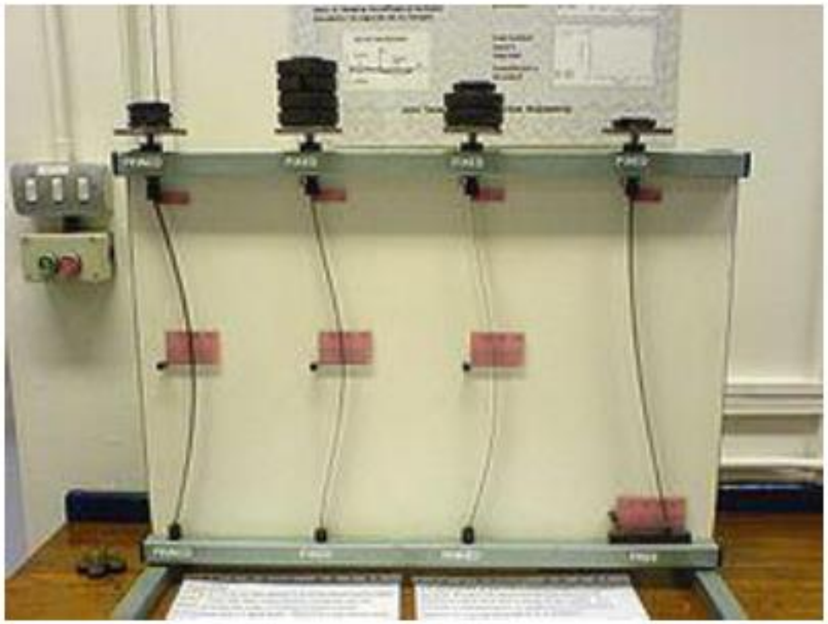
The instability "buckling" of the specimen under compressive stresses

Buckling is characterized by a sudden failure of a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. This mode of failure is also described as failure due to elastic instability.



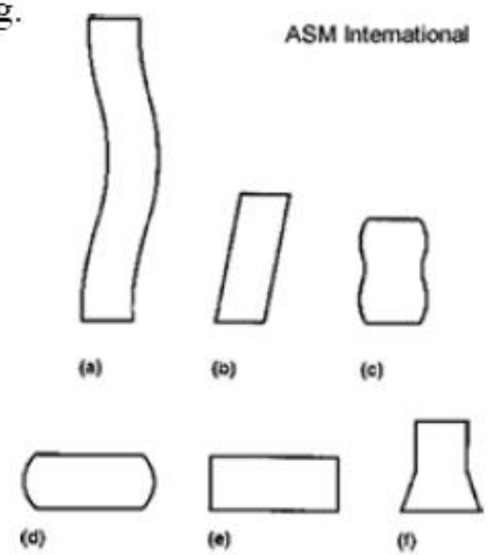
Mathematical analysis of buckling makes use of an axial load eccentricity that introduces a moment, which does not form part of the primary forces to which the member is subjected. When load is constantly being applied on a member, such as specimen, it will ultimately become large enough to cause the member to become unstable. Further load will cause significant and somewhat unpredictable deformations, possibly leading to complete loss of load-carrying capacity. The member is said to have buckled, to have deformed.

Difficulties of The Compression test



A demonstration model illustrating the different "Euler" buckling modes. The model shows how the boundary conditions affect the critical load of a slender column. Notice that each of the columns are identical, apart from the boundary conditions.

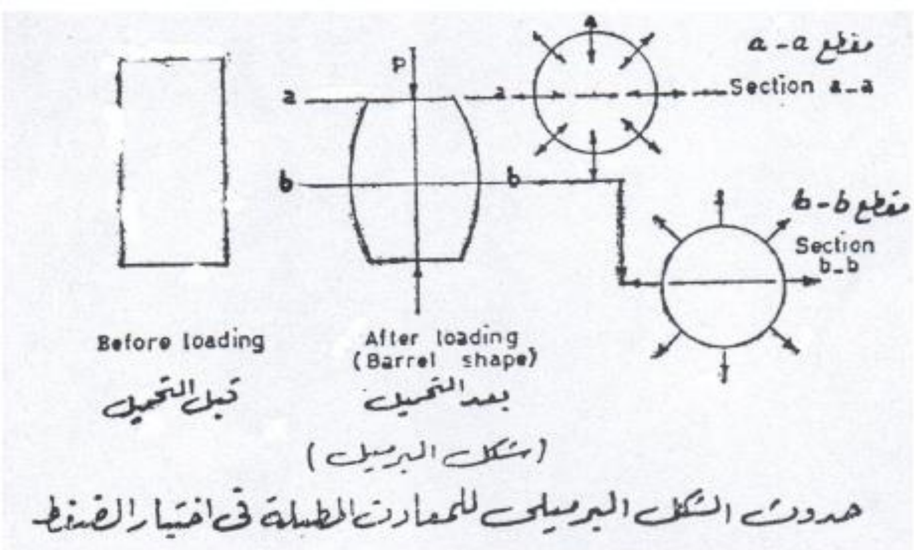
A short specimen under the action of an axial load will fail by direct compression before it buckles, but a long column loaded in the same manner will fail by buckling (bending), the buckling effect being so large that the effect of the direct load may be neglected. The intermediate-length column will fail by a combination of direct compressive stress and bending.

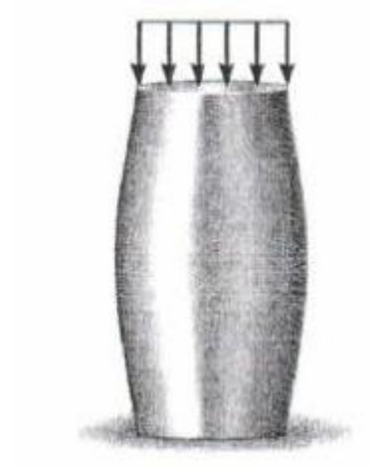


Difficulties of The Compression test

The friction forces near specimen ends

As the load applied on the specimen increases the specimen will generally bulge out or become barrel shaped as the strains become larger due to existence of friction force at both ends that prevents the specimen from freely expand near both ends while the specimen expands freely near the mid-height. Thus the specimen takes the barrel shapes.





Compression causes material to bulge out

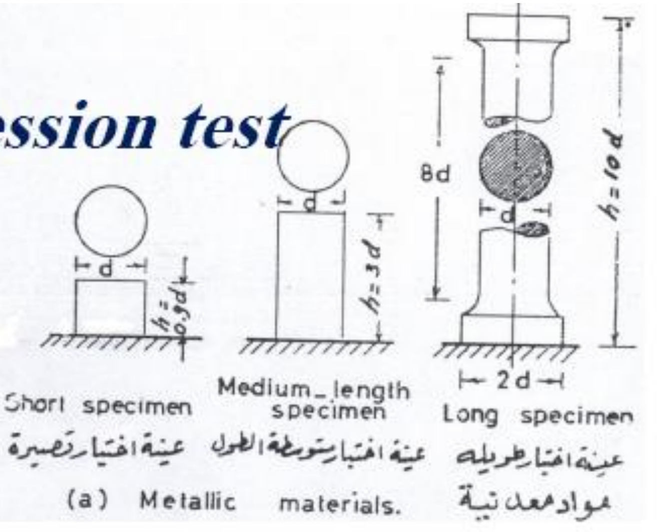
Difficulties of The Compression test

The size of the specimen

One way to prevent buckling is to increase the cross sectional area of the specimen. Due to the high compressive strength, the required applied load will be high and requires testing machines with higher capacity.

Specimen dimensions

Standard Specimens in Compression test

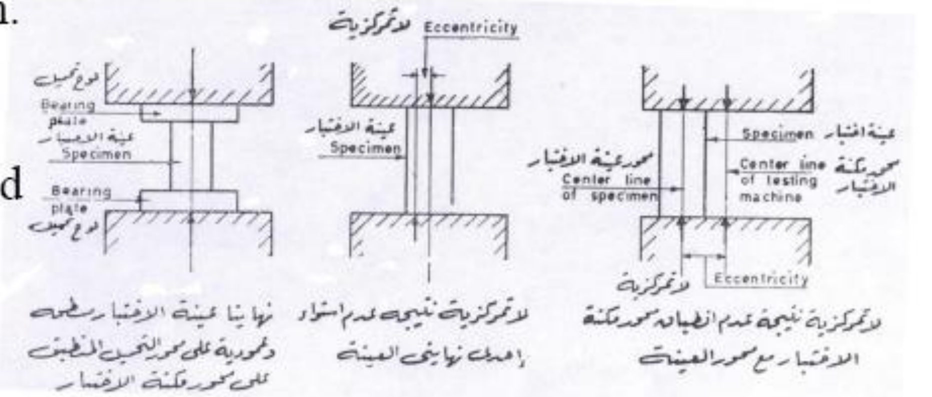


Standard Specimens:

Long Specimens: h = (8-10) d
Used to draw load-deformation curve.
Medium Length Specimens h= (3-5) d
Used for determining the compressive strength.
Short Specimens: h = (0.9-1) d
Used for determining the effect of friction force of compressive strength.

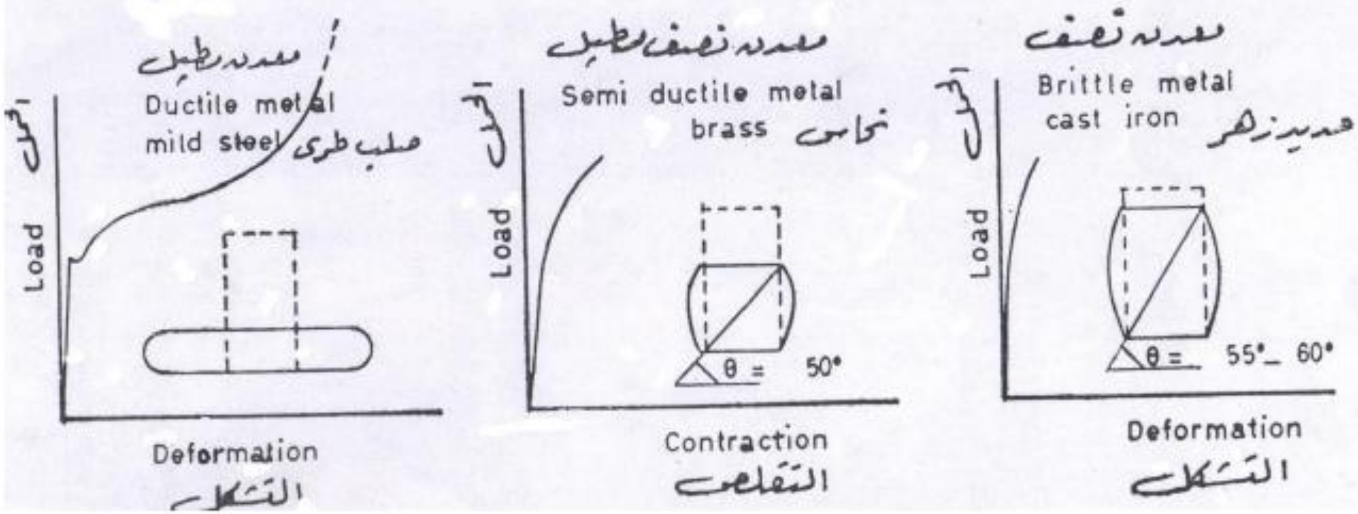
General Requirements:

Eccentric loading should be prevented by ensuring both ends should be ***smooth***, ***flat***, ***parallel to each other*** and ***perpendicular*** to the specimen axis.



Behavior of metals under compressive loading

Standard specimens with circular cross section subjected to axial compressive will suffer from excessive contraction “deformation” under the loading.
All metals will have a proportional line at the beginning.
Only ductile metals will exhibit yielding.
Brittle (cast iron) and Semi-ductile (brass) metals will take the barrel shape and fails due to shear stresses on a plane inclined by 45+ϕ/2, where ϕ:is the angle of internal friction.
Failure occurs at the ultimate load.

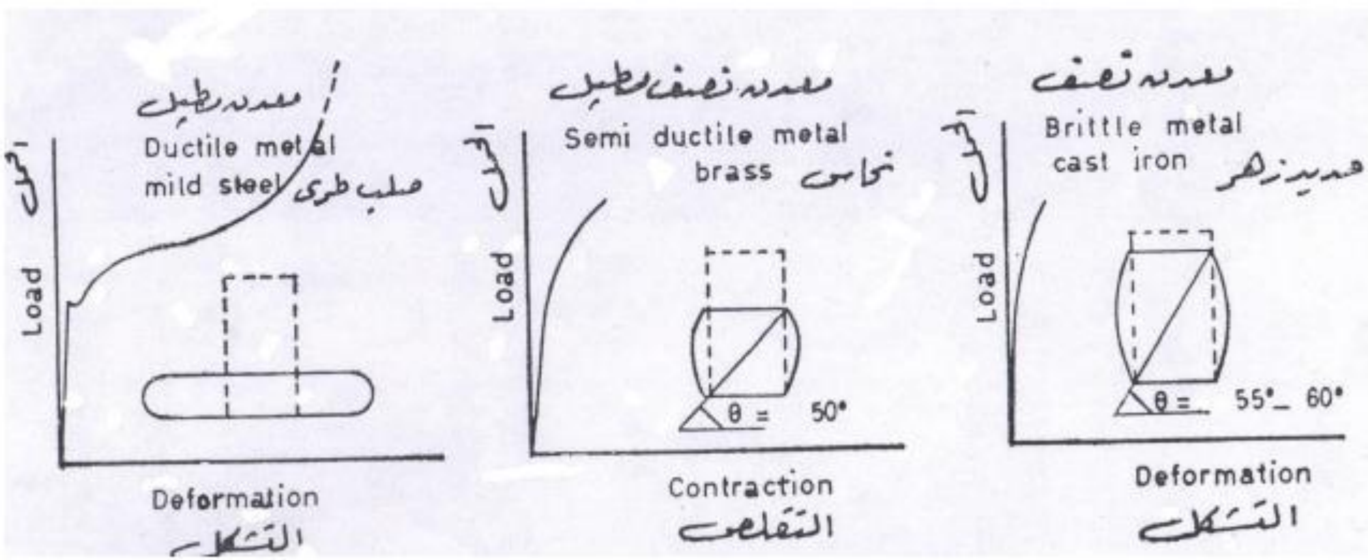


Behavior of metals under compressive loading

Brittle (cast iron) metals will fail due to shear stresses on a plane inclined by (55-60) deg.

Semi-ductile (brass) metals will fail due to shear stresses on a plane inclined 50 deg.

Ductile metals will take the barrel shape ***But*** will not fail and the specimen keeps on flattening under compressive stresses. At rare cases ductile metals will fail in vertical cracks due to existence of impurities in the ductile metal.

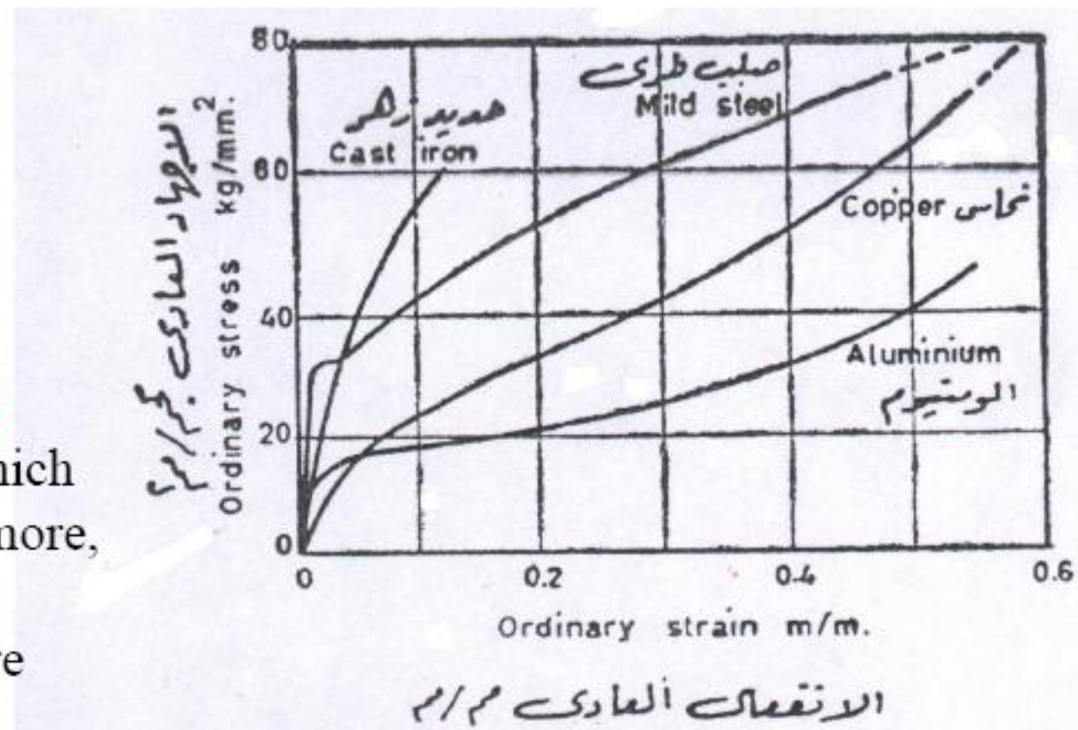


Behavior of metals under compressive loading

- Compressive stress and strain, are calculated similar to tensile counterparts respectively.

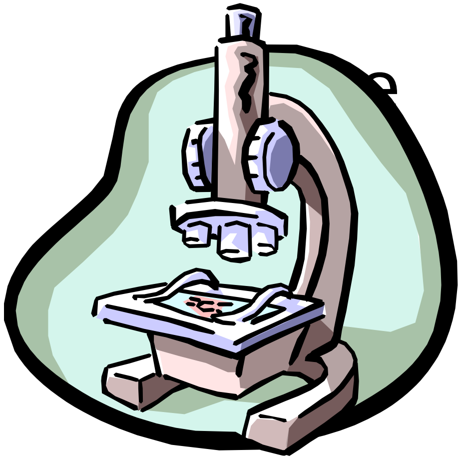
$$\sigma = \frac{P}{A_0} \qquad \varepsilon = \frac{\Delta L}{L_0}$$

- By convention, a ***compressive force*** is taken to be ***negative***, which yields a negative stress. Furthermore, since L_o is greater than L_i, compressive strains computed are necessarily negative.



Observations in Compression Tests

- **Observations:** change in dimensions, critical loads, type of failure,…
- **Brittle** materials: rupture either along a diagonal plane, or with a cone (for cylindrical specimens) or a pyramidal (for square specimens) shaped fracture.
- **Ductile** materials: bulge laterally, e a barrel shape.



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The cast iron under tension & compression

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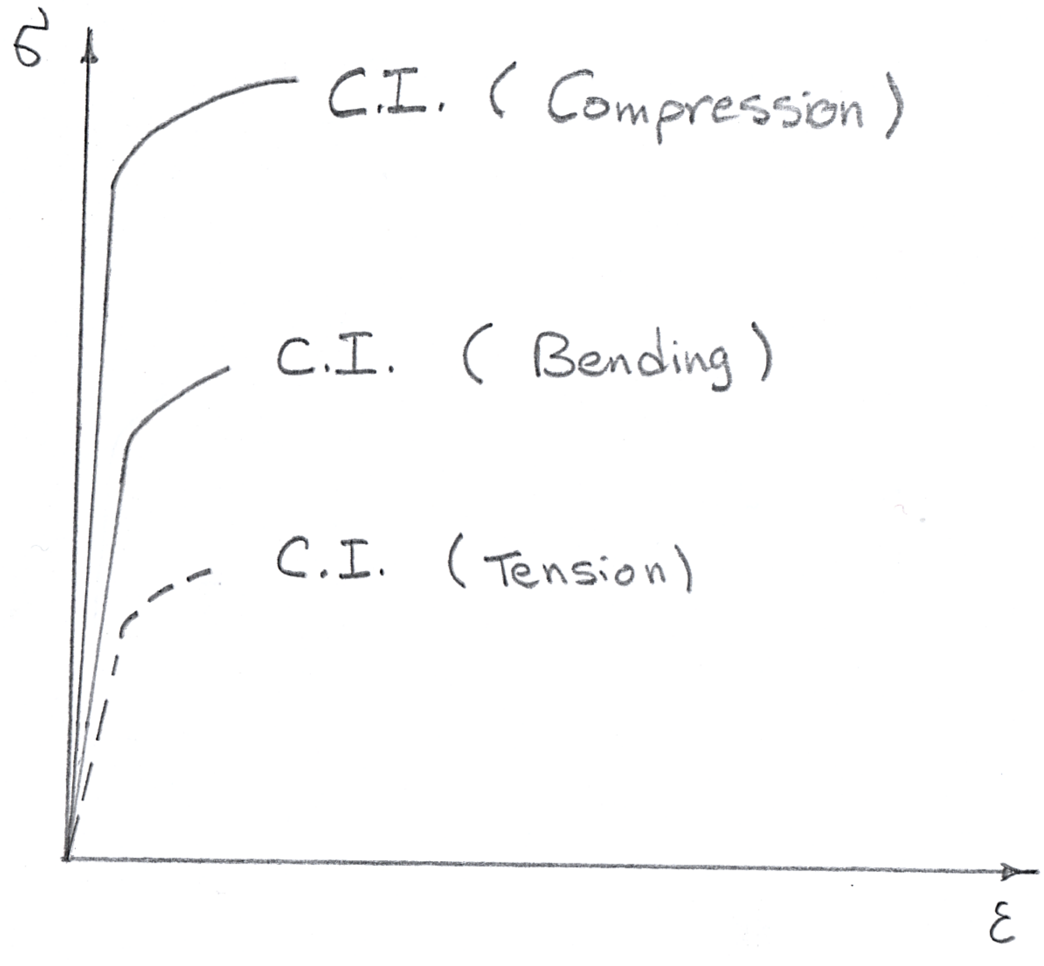
Effect of load direction on the mechanical properties of C.I.



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Stress distribution over the cross section under tension/compression loading

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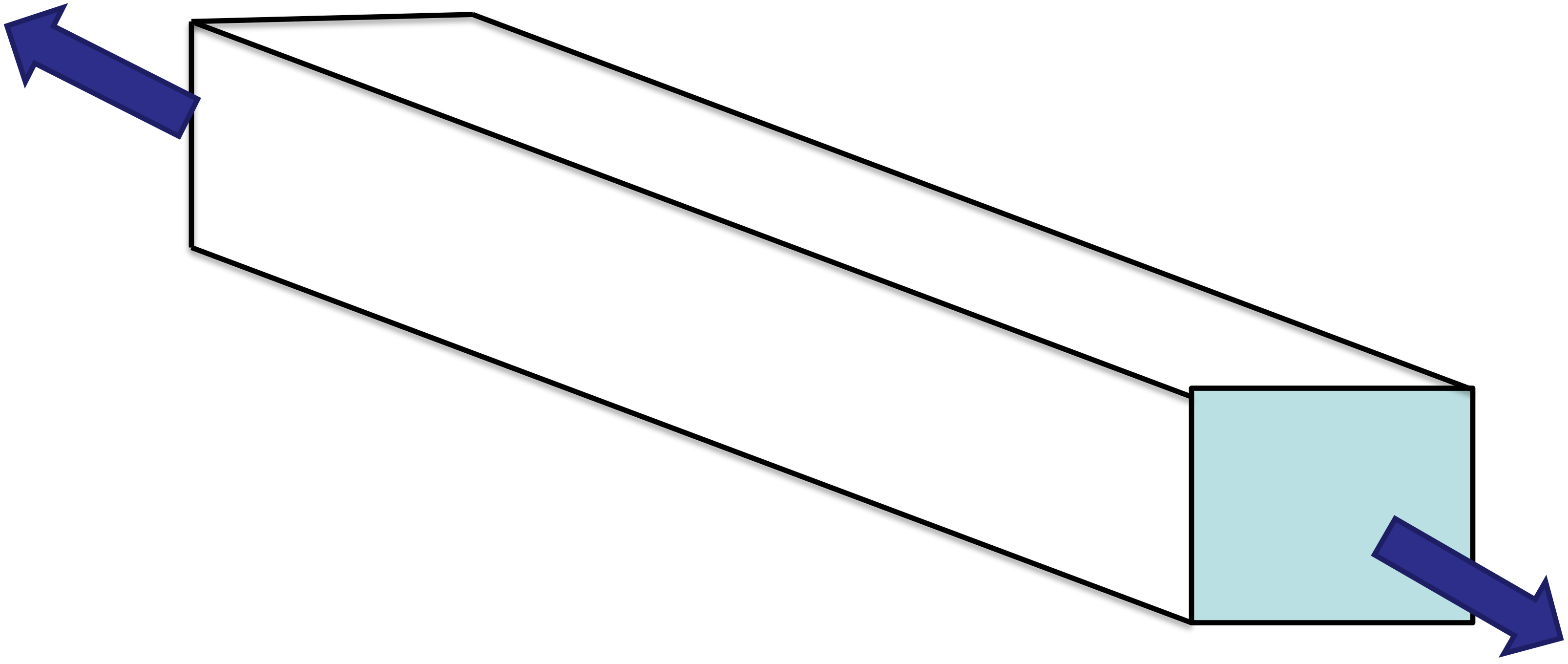
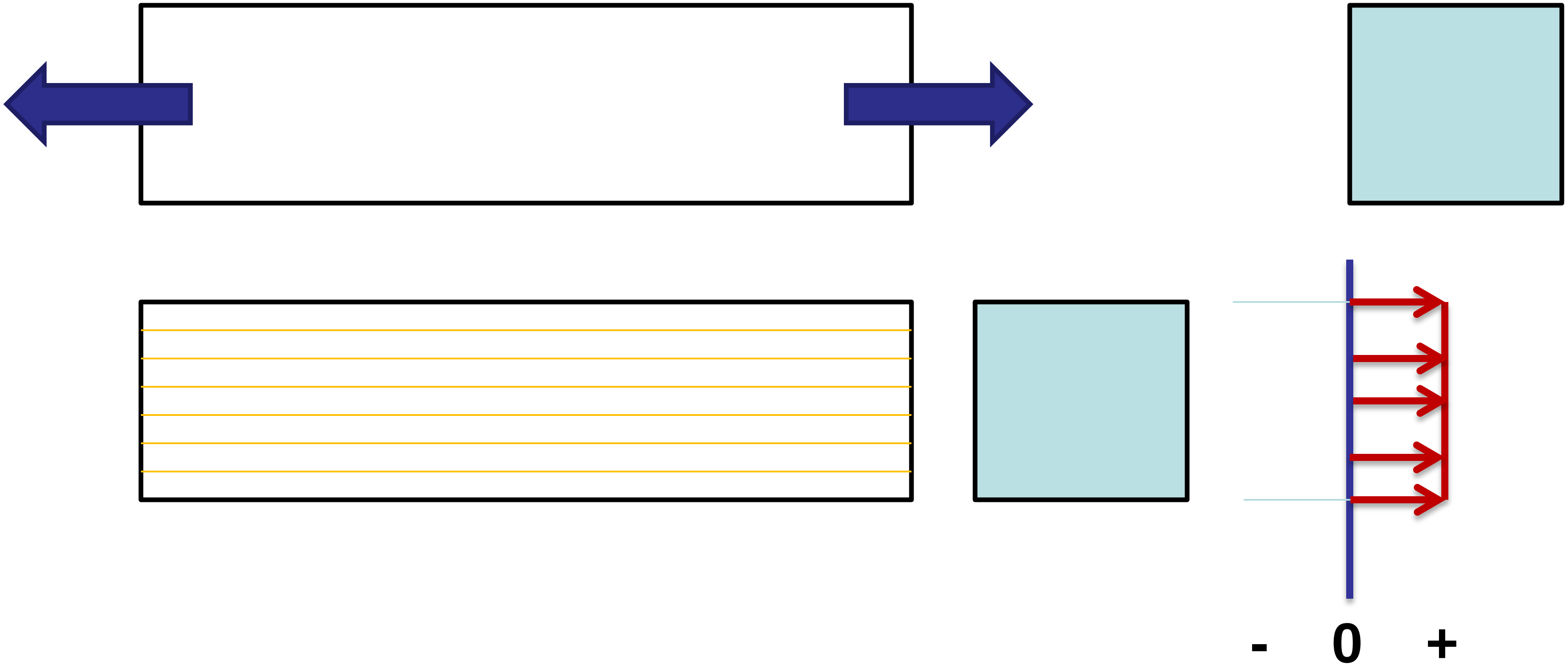
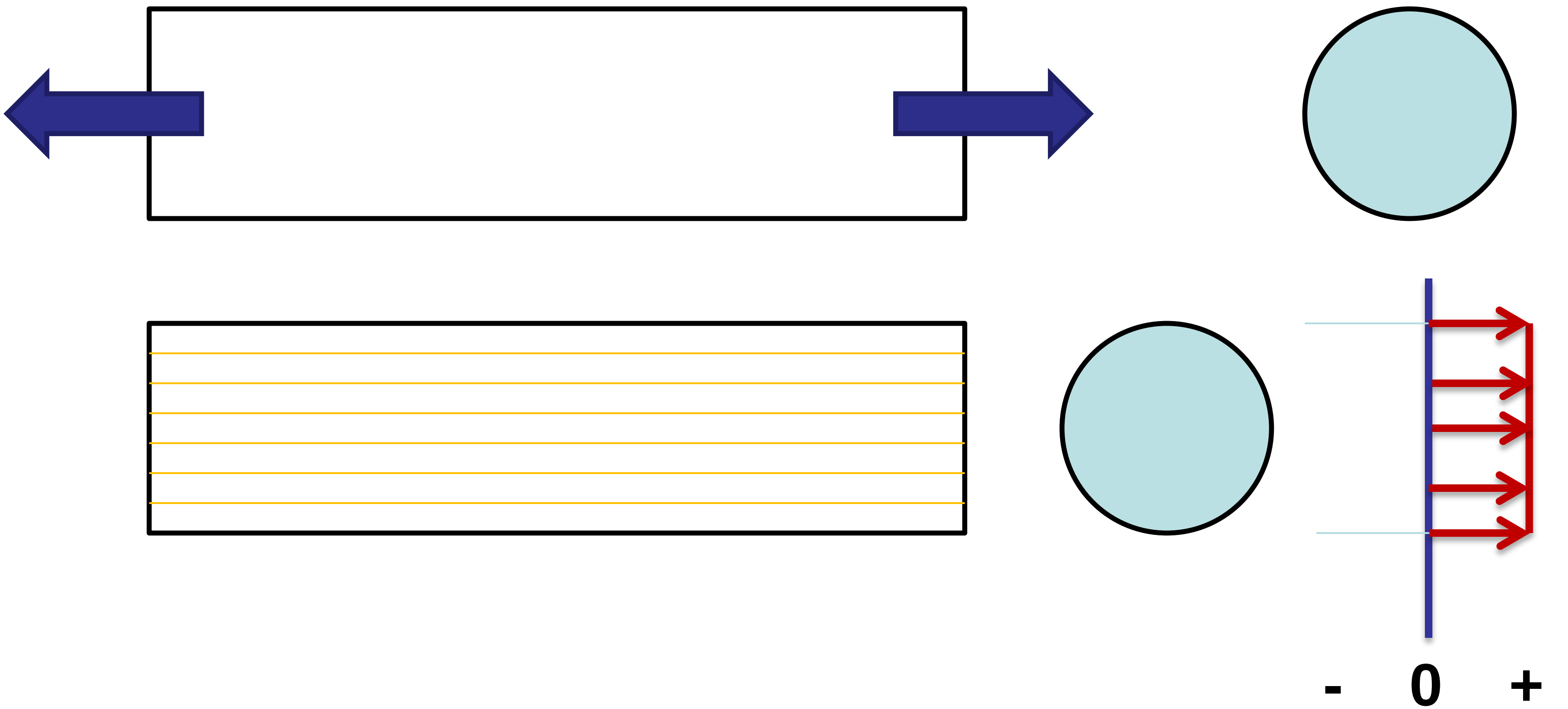
Stress distribution over the cross section



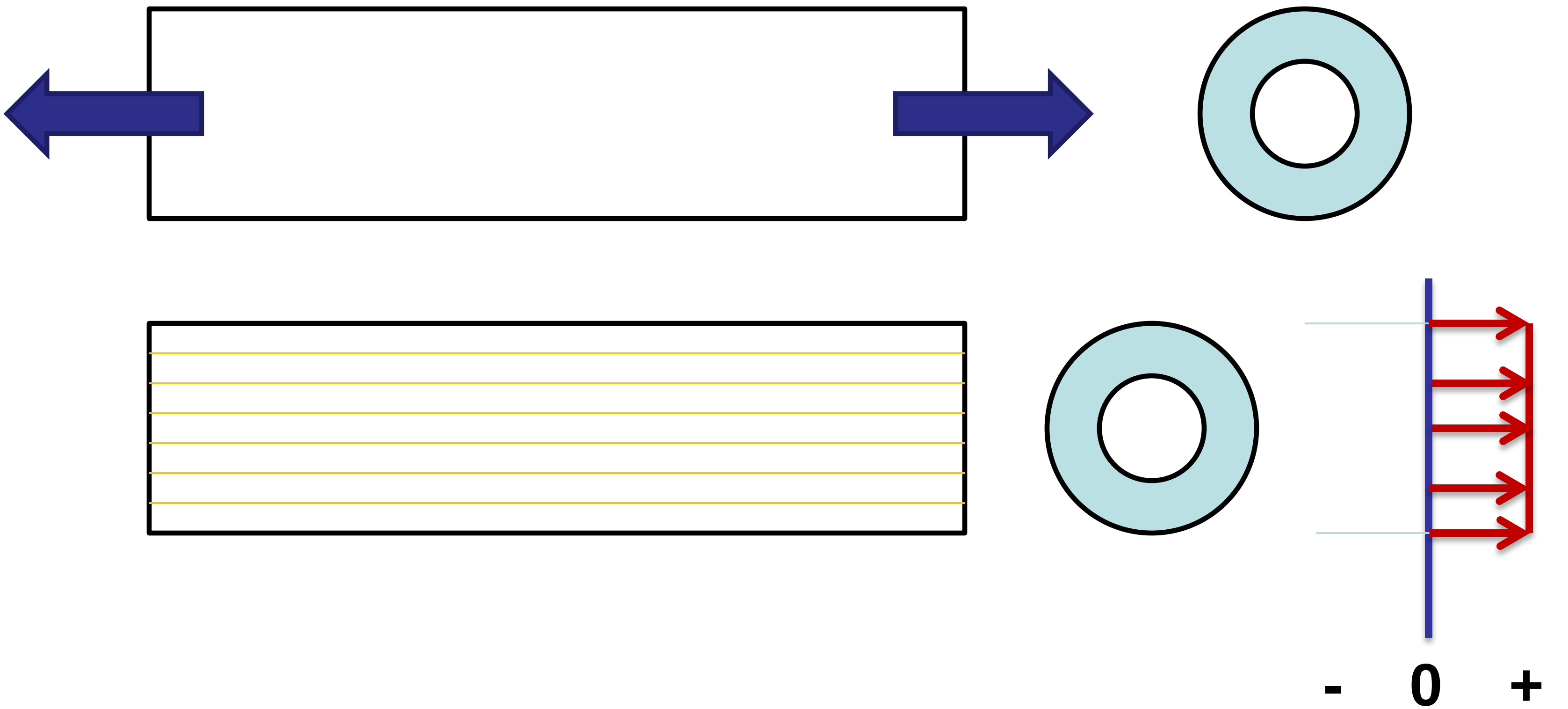
Stress distribution over the cross section



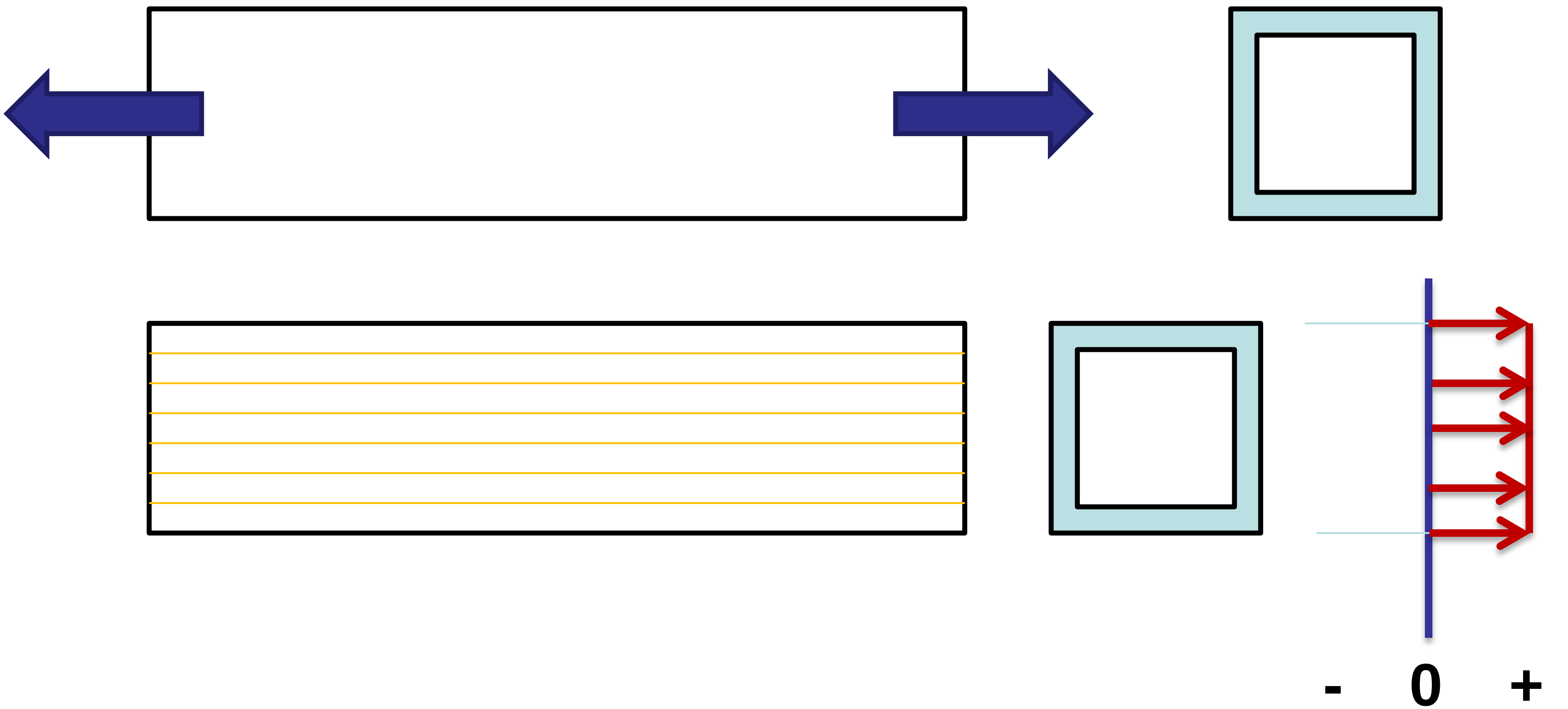
Stress distribution over the cross section



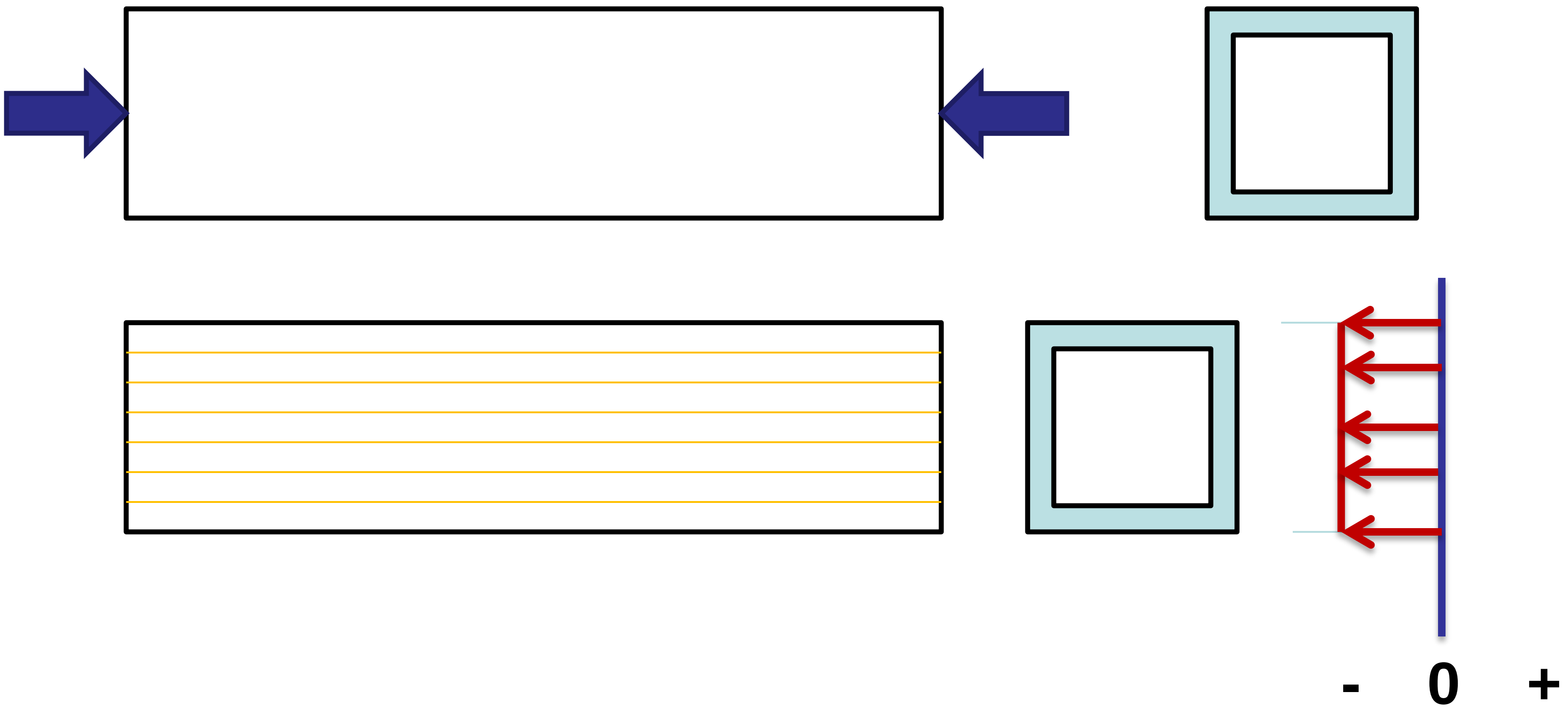
Stress distribution over the cross section



Stress distribution over the cross section



Stress distribution over the cross section



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Theory of Simple Bending

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Introduction

The behaviour of beams under bending is so complex.

The stress, strain, dimension, curvature, elasticity, are all related, under certain assumption, by *the theory of simple bending*. This theory relates to beam flexure resulting from couples applied to the beam without consideration of the shearing forces.

Notation

These notation are used in the next slides:

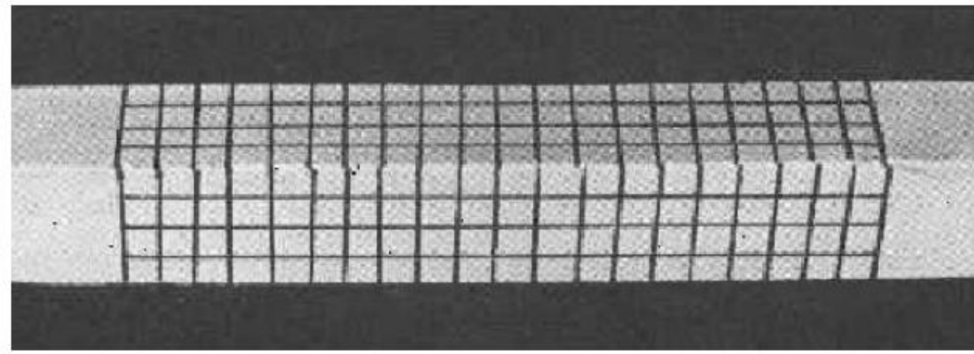
ε = strain
E = Young's Modulus = σ /e (N/m^2)
y = distance of surface from neutral surface (m).
ρ = Radius of neutral axis (m).
I = Moment of Inertia (m^4 - more normally cm^4)
Z = section modulus = I/y_{max}(m^3 - more normally cm^3)
F = Force (N)
x = Distance along beam
δ = deflection (m)
θ = Slope (radians)
σ = stress (N/m^2)

Beam Bending Theory

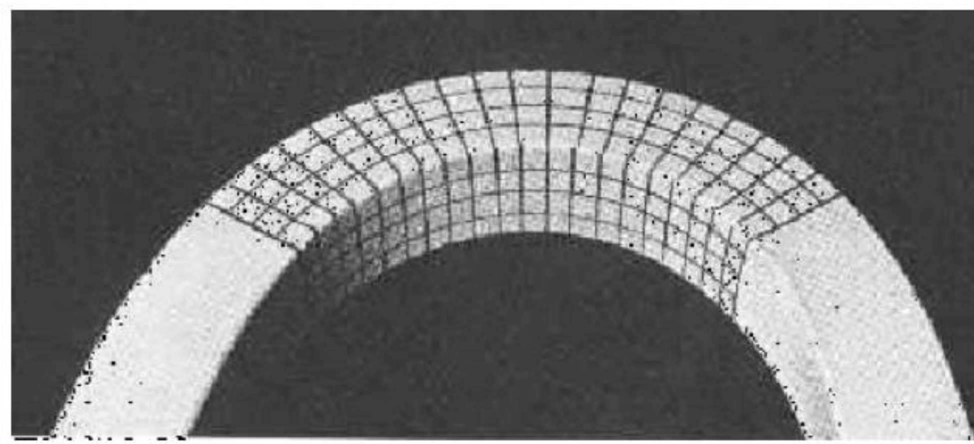
Consider the sponge beam shown:

Plane sections before bending were parallel to each other and perpendicular to the centre line of the beam.

Plane sections REMAIN plane and perpendicular to the centre line of the beam after bending.

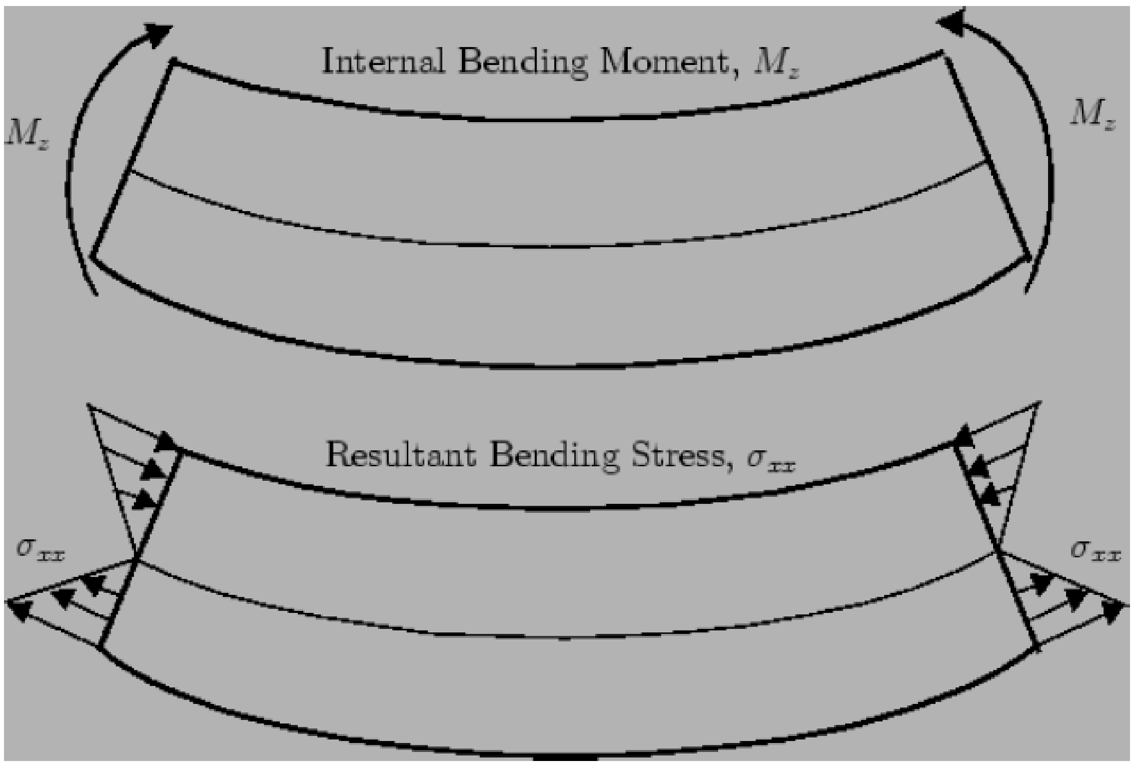


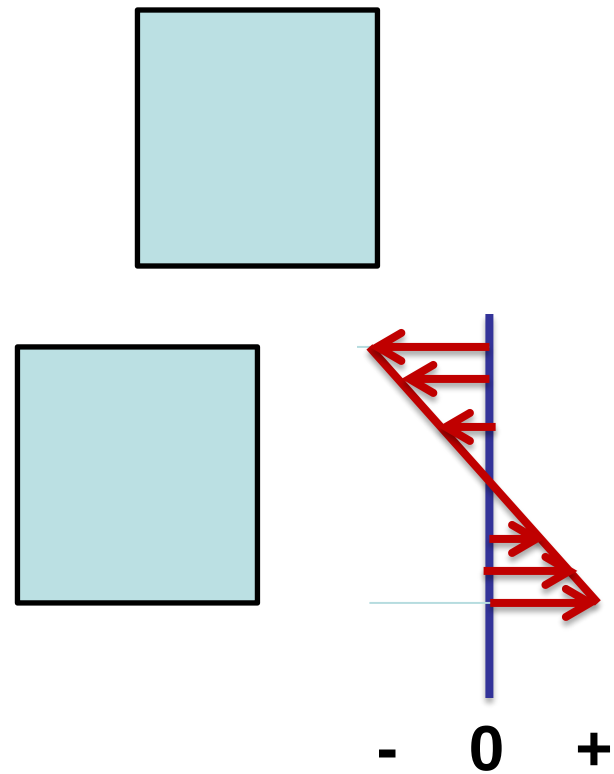
Before bending



After bending

Stress distribution over the cross section





Theory of Simple Bending

A straight bar of homogeneous material is subject to only a moment at one end and an equal and opposite moment at the other end.

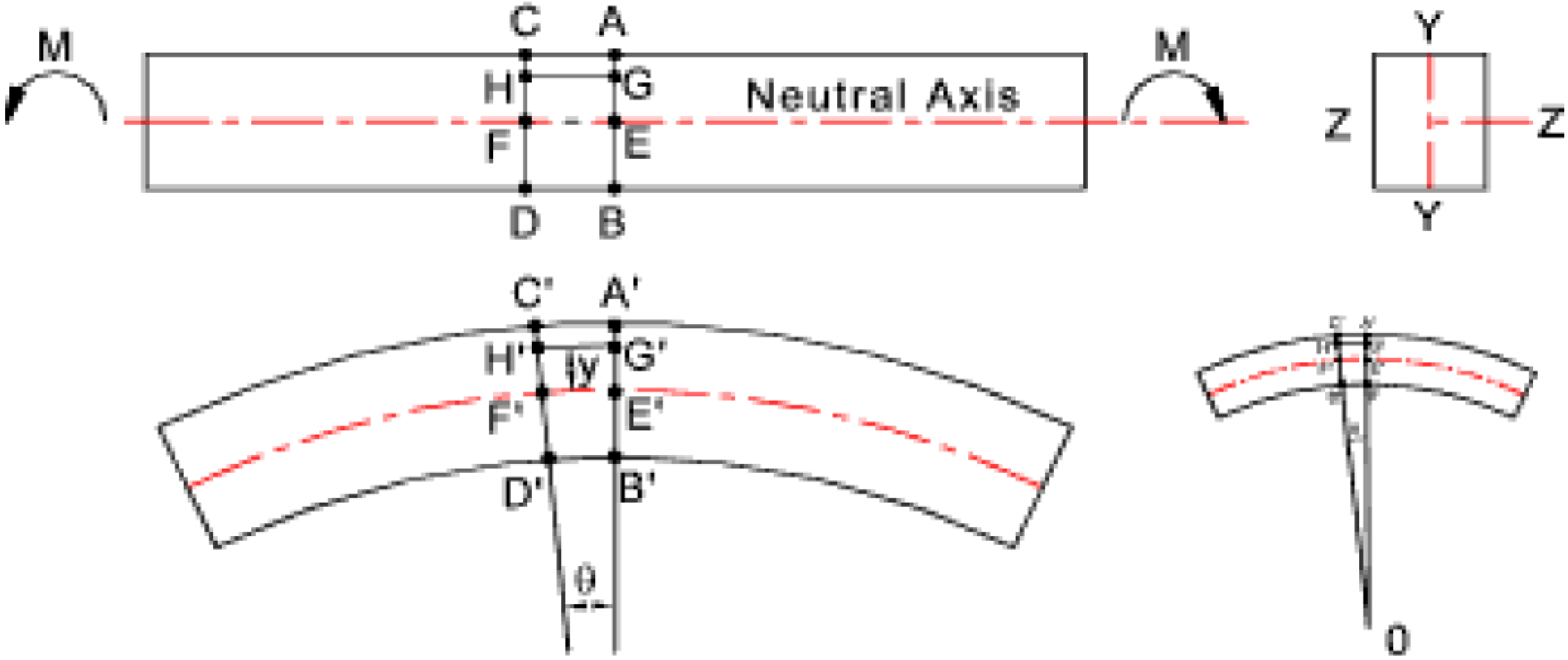
Assumptions

1.The beam is symmetrical about Y-Y.

2.The traverse plane sections remain plane and normal to the longitudinal fibers after bending (Beroulli's assumption).

3.The fixed relationship between stress and strain (Young's Modulus)for the beam material is the same for tension and compression ($\sigma = E.e$).

Theory of Simple Bending



Strains in Beams

1. Consider two section very close together (AB and CD).
2. After bending the sections will be at A'B' and C'D' and are no longer parallel.
3. AC will have extended to A'C' and BD will have compressed to B'D'.
4. The line EF will be located such that it will not change in length.
5. This surface is called neutral surface and its intersection with Z_Z is called the neutral axis.
6. The development lines of A'B' and C'D' intersect at a point 0 at an angle of θ radians and the radius of E'F' = ρ
7. Let y be the distance(E'G') of any layer H'G' originally parallel to EF.

$$H'G'/E'F' = (\rho + y)\theta \,/ R\,\theta = (\rho + y)/\,\rho$$

8. And the strain ε at layer H'G' =

$$\varepsilon = (H'G' - HG) / HG = (H'G' - HG) / EF =$$

$$\varepsilon = [(\rho + y)\theta - \rho\,\theta] / \rho\,\theta$$

$$\varepsilon = y/\rho$$

Stresses in Beams

The accepted relationship between stress and strain is

$$\sigma = E.\varepsilon$$

Therefore,

$$\sigma = E.\varepsilon = E.\ y/\rho$$

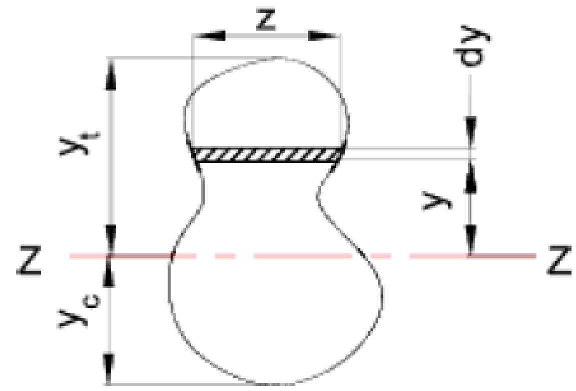
$$\sigma / E = y / \rho$$

$$\sigma = E.y / \rho \qquad or \qquad \underline{\boldsymbol{\sigma / y = E / \rho}}$$

Therefore, for the illustrated example, the tensile stress is directly related to the distance below the neutral axis. The compressive stress is also directly related to the distance above the neutral axis.
Assuming E is the same for compression and tension the relationship is the same.

Neutral axis

As the beam is in static equilibrium and is only subject to moments (no vertical shear forces) the forces across the section (AB) are entirely longitudinal and the total compressive forces must balance the total tensile forces.



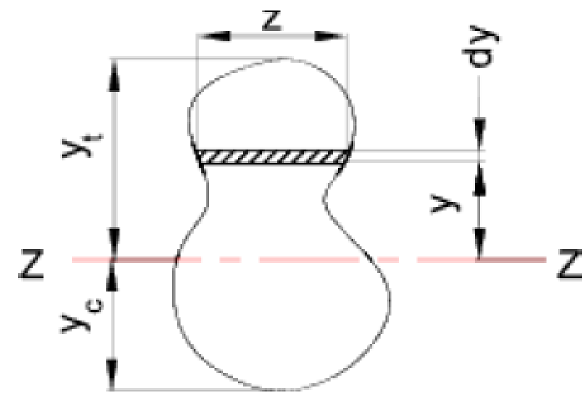
The internal couple resulting from the sum of (σ.dA .y) over the whole section must equal the externally applied moment.

$$\sum(\sigma.\delta A) = 0 \quad \text{therefore} \quad \sum(\sigma.z\ \delta y) = 0$$

$$\text{As} \quad \sigma = \frac{yE}{R} \quad \text{therefore} \quad \frac{E}{R}\sum(y.\ \delta A) = 0 \quad \text{and} \quad \frac{E}{R}\sum(y.\ z\delta y) = 0$$

This can only be correct if Σ(yδa) or Σ(y.z.δy) is the ***moment of area of the section*** about the ***neutral axis***. This can only be zero if the axis passes through the centre of gravity (***centroid***) of the section.

Bending Moment "Elastic Case"



The internal couple resulting from the sum of (σ.dA .y) over the whole section must equal the externally applied moment.

Therefore the couple of the force resulting from the stress on each area when totaled over the whole area will equal the applied moment.

The force on each area element = $\sigma.\,\delta A = \sigma.\,z.\,\delta y$

The resulting moment = $y.\sigma.\,\delta A = \sigma.\,z.\,y.\,\delta y$

The total moment $M = \sum(y.\sigma.\delta A)$ and $\sum(\sigma.z.y\,\delta y)$

Using $\frac{E}{R}y = \sigma$

$M = \frac{E}{R}\sum(y^2.\,\delta A)$ and $\frac{E}{R}\sum(z.y^2\delta y)$

$\sum(y^2.\delta A)$ is the Moment of Inertia of the section(I)

From the above the following important simple beam bending relationship results

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

Bending Moment "Elastic Case"

$$\frac{M}{I} = \frac{E}{\rho} = \frac{\sigma}{y}$$

M – Bending moment or Moment may vary depending on the load example

I – Moment of Inertia.

$I = bd^3/12$ for rectangular section and $y = d/2$

$I = \pi(D^4_o - D^4_i)/64$ for hollow pipe and $y = D_o/2$

σ – Stress due to bending moment.

E – Modulus of Elasticity or Young's modulus.

ρ- Radius of curvature due to bending.

y – distance measured from section centroid

Bending Stress "Elastic –Plastic Case"

The stress distribution beyond elastic limit may not be linear since it follows the stress strain curve under either tensile or compressive stresses respectively. In addition, the relation between stresses and strains may not be linear as well. There is no definite relation for bending stresses beyond yielding up until this time. The bending stresses should be calculated on a case-by case basis.

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Bending test

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Beam Bending Test

1. Extremely low ductility "brittle materials" does not allow measuring their mechanical properties (specially strength and stiffness) accurately by conventional tensile test, which is widely used for metals.
2. Brittle Materials, including cast iron, concrete, wood, ceramics, are tested Flexure Test by (Transverse Beam Bending Test).
3. There are two standard Flexure Test methods:

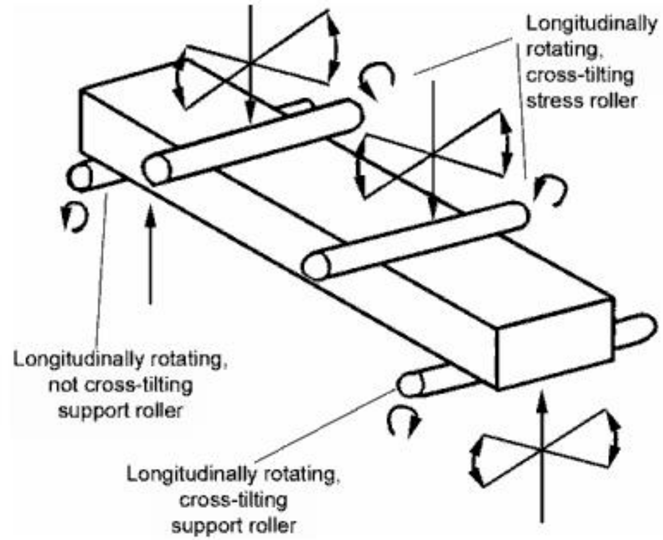
3-point Flexure Test **or** **4-point Flexure Test**

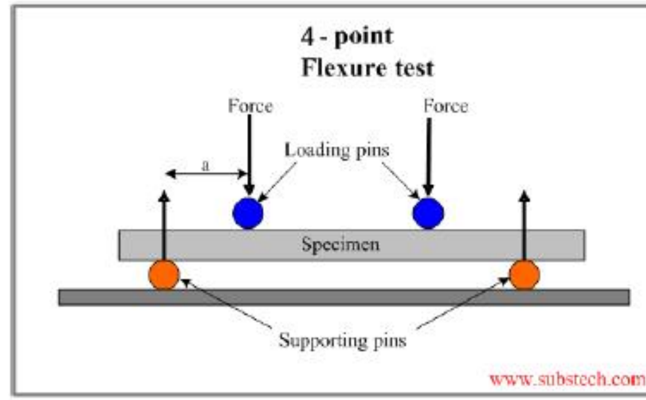
In this test a specimen with round, rectangular or flat cross-section is placed on two parallel supporting pins as follows;

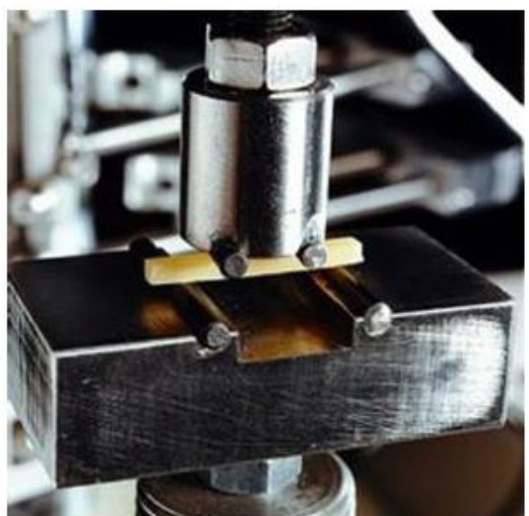
1. The supporting and loading pins are mounted in a way, allowing their free rotation about:
2. axis parallel to the pin axis;
3. axis parallel to the specimen axis.
4. This configuration provides uniform loading of the specimen and prevents friction between the specimen and the supporting pins.

4-point Flexure Test

In this test the loading force is applied by means of two loading pins with a distance between them equal to one-third of the distance between the supporting pins (span). The test provides at least the middle half of the specimen between the applied loads to be subjected to constant bending moment and zero shear. Yet the test is slightly more complicated.



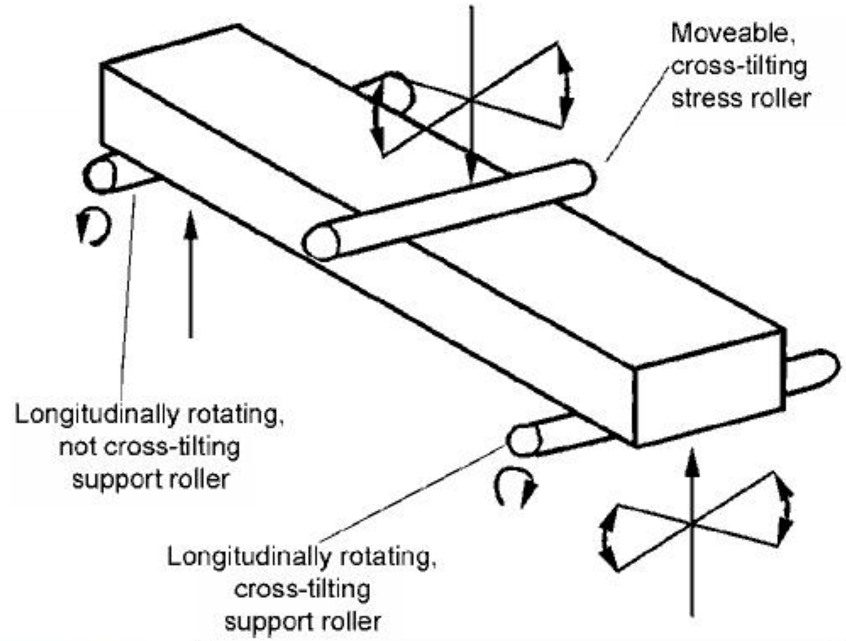


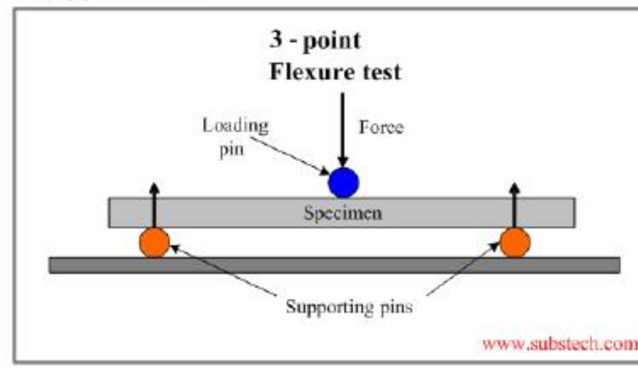


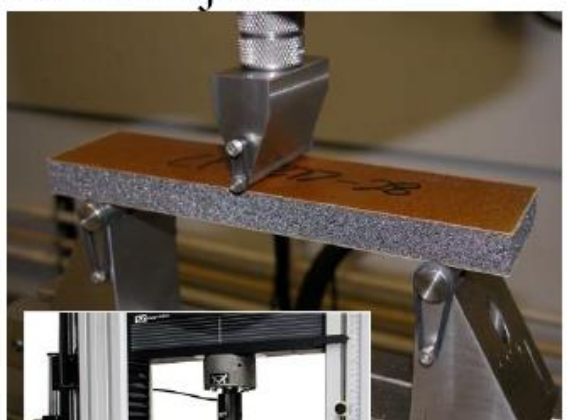
3-point Flexure Test

The loading force is applied in the middle (mid-span) by means of single loading pin.

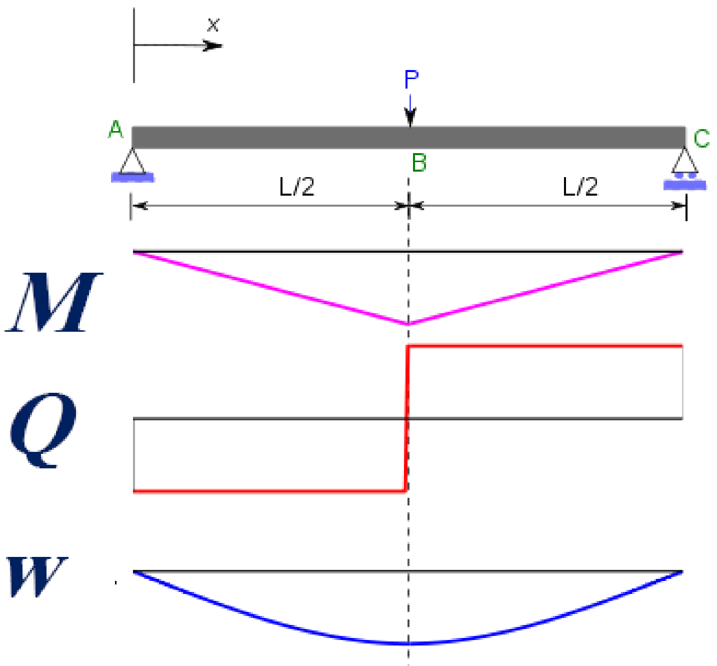
The test is much simpler to be carried out. Yet, only one single section is subjected to maximum bending moment in the whole specimen.



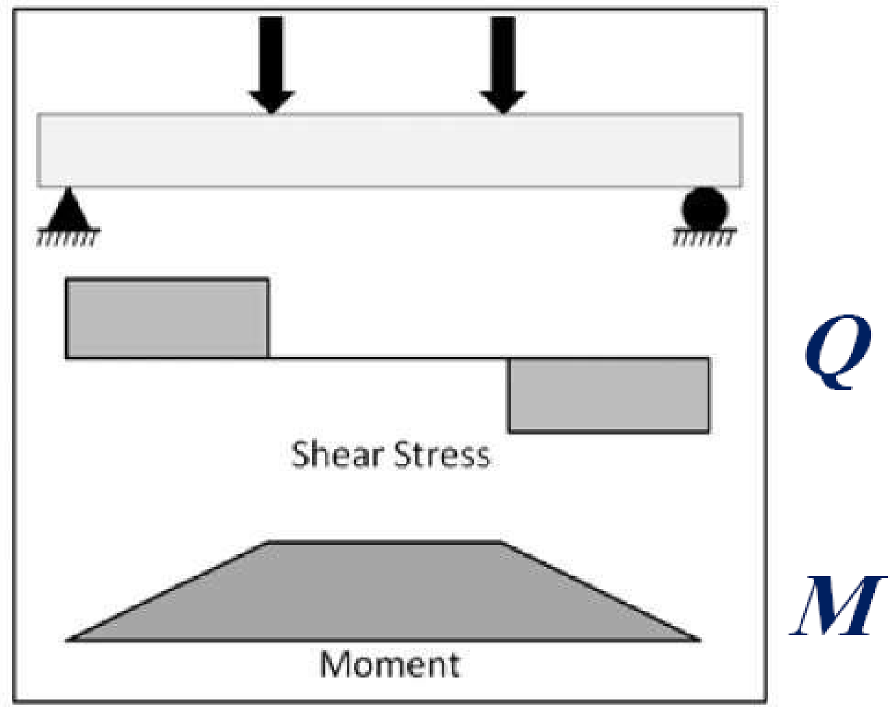




3-point Flexure Test



4-point Flexure Test



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The mechanical properties after bending test

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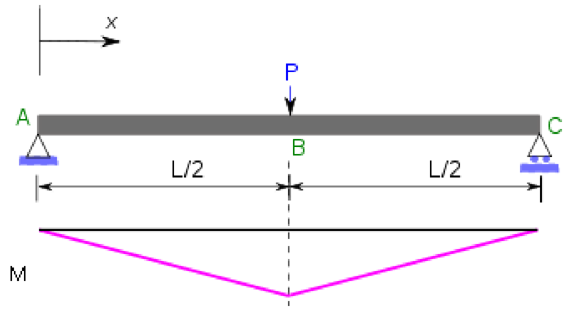
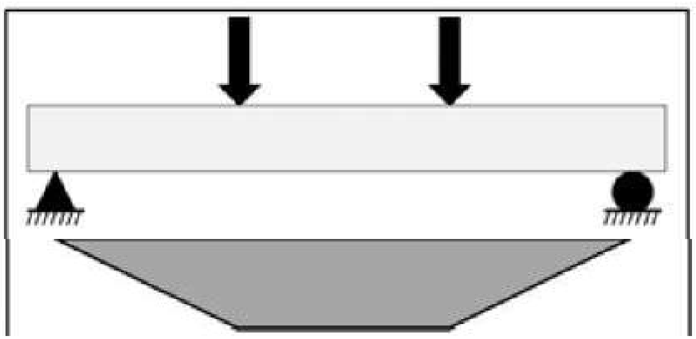
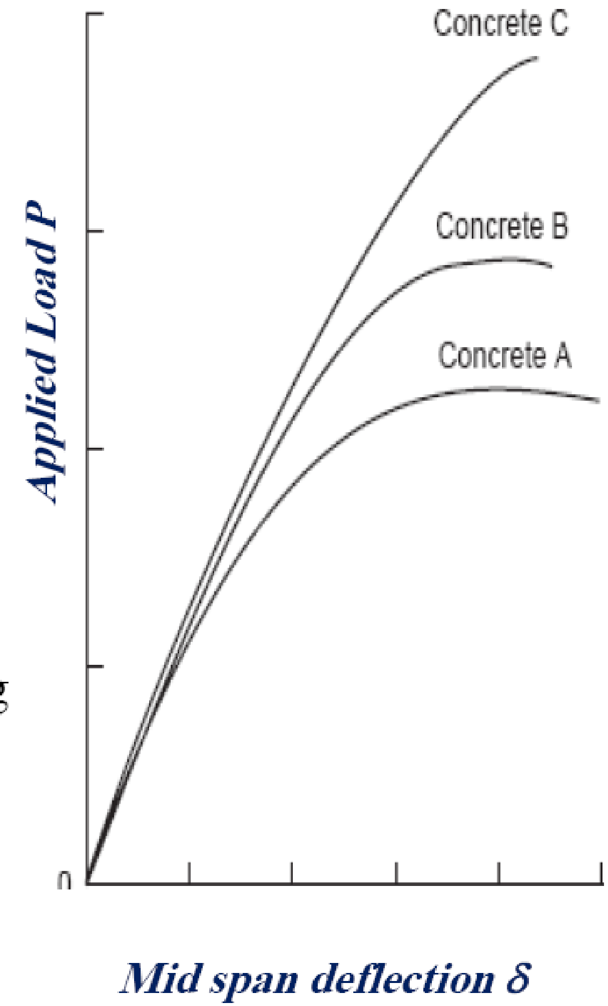
Mechanical Properties in Flexure





Dial Strain Gauge is used to measure the mid span deflection δ



As a result of the loading, the specimen bends, causing deformation, tension stresses in its convex side and compression stress in the concave side.

The applied load and the mid span deflection are measured for every load value. The results are then plotted in the load-deflection diagram.

Proportional Limit

Proportional limit is the maximum stress where stress is proportional to the strain.

Proportional limit load is defined as the end of the straight line. Thus, if the proportional limit load is P_{PL} and the original cross sectional area = Ao, then:

Maximum bending moment @ Prolortional limit = $\boldsymbol{M_{PL}}$

Proportional Limit = $\sigma_{PL} = \boldsymbol{M_{PL} * Y_{max} / I_x}$

$M_{PL} = P_{PL} * L/4$ *(for 3-point beam bending test)*

Y_{max} = h/2 for rectangular or square cross sections.

= $D_o/2$ for circular cross sections.

I_x = $bh^3/12$ for rectangular section

I_x = $\pi(D^4_o - D^4_i)/64$ for hollow pipe.

Stiffness "Young's Modulus"

The load deformation diagram for most engineering materials exhibit a linear relationship between applied load and deformation within the elastic region. Consequently, an increase in stress causes a proportionate increase in strain. The relation between *the modulus of elasticity or Young's modulus*, Applied load, measured deflection and the beam property is give as follows;

$$\delta_{PL} = \frac{P_{PL} L^3}{48EI}$$

Modulus of Rupture

<u>Modulus of Rupture (Flexural Strength)</u> is the stress of the extreme fiber of a specimen at its failure in the Flexure Test.

Modulus of rupture is calculated using the same formulae for simplicity and since there is no exact solution until now :

Maximum bending moment @ maximum load = $\boldsymbol{M_{max}}$

Proportional Limit = MoR = $\boldsymbol{M_{max} * Y_{max} / I_x}$

$M_{max} = P_{max} * L/4$ *(for 3-point beam bending test)*

Y_{max} = h/2 for rectangular or square cross sections.

= $D_o/2$ for circular cross sections.

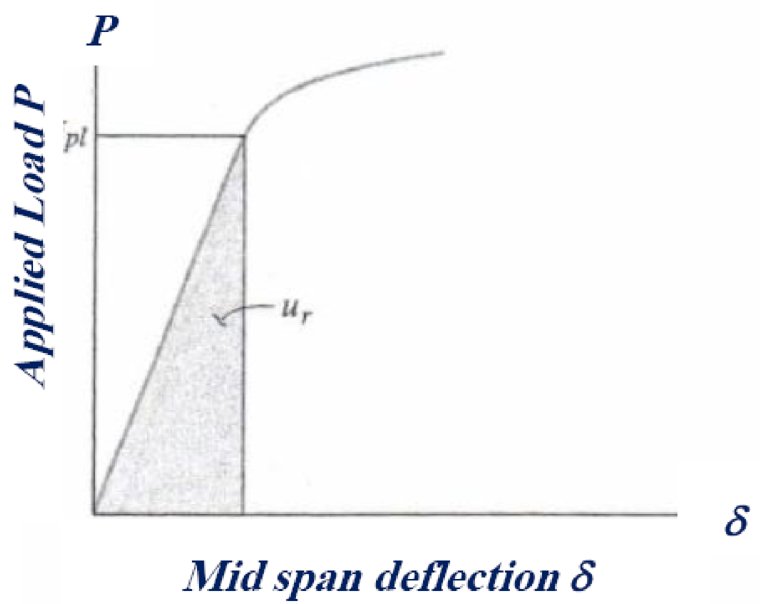
I_x = $bh^3/12$ for rectangular section

I_x = $\pi(D^4_o - D^4_i)/64$ for hollow pipe.

Modulus of Resilience

A material's resilience represents the ability of the material to absorb energy without any permanent damage to the material. In particular, when the load a reaches the proportional limit, the strain-energy density, is calculated by and is referred to as the *modulus of resilience Ur*. Mathematically it is the area under the straight line "elastic region" of the load-deformation curve per unit volume.

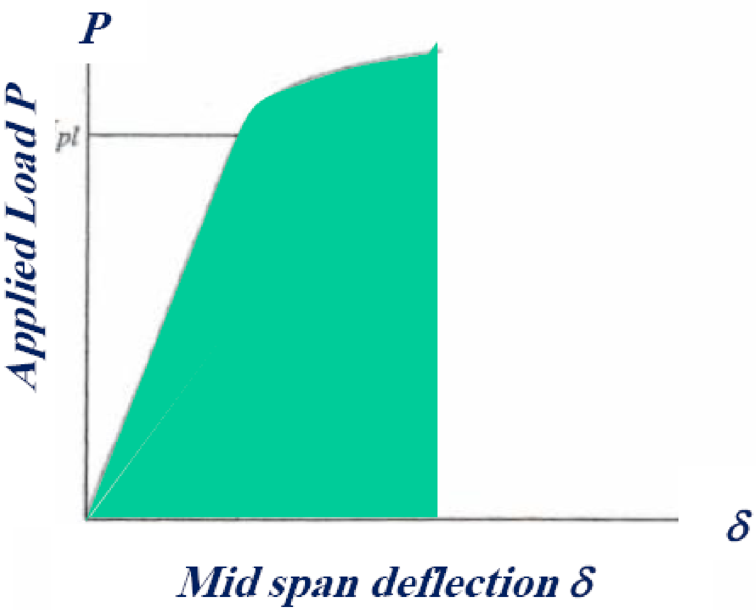
$$U_r = \frac{P_{PL} * \delta_{PL}}{2 * A * L}$$



Modulus of Toughness

Another important property of a material is *the modulus of toughness*, Ut. This quantity represents the entire area under the stress-strain diagram, and therefore it indicates the strain energy density of the material just before it fractures.

$$U_t = \frac{2 * P_{max} * \delta_{max}}{3 * A * L}$$



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13.04.2019 – Week 10

Watching bending testing practice

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Cold Bent Test

Dr. Mahmoud Khedr

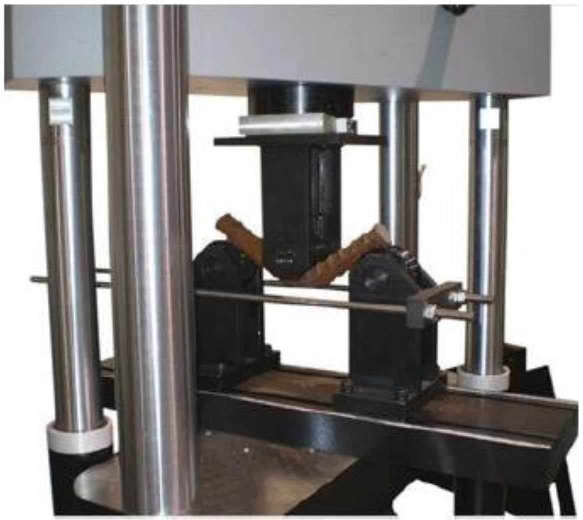
Cold Bent Test

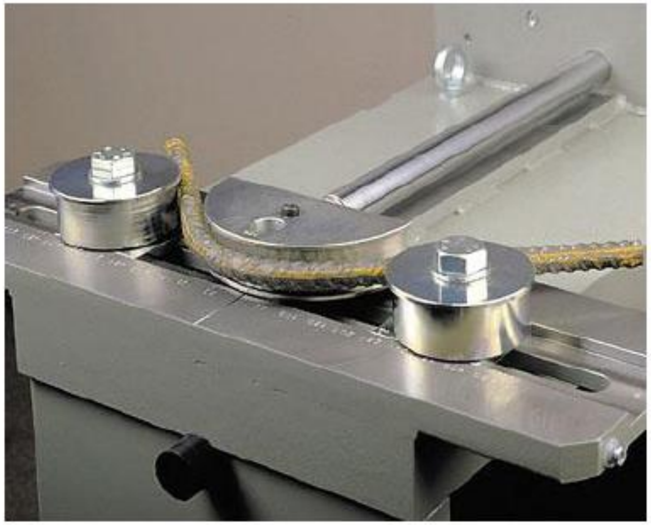
Rebar is bent into a multitude of different shapes to reinforce concrete structures. To ensure that the material is capable of being bent without significant ***strength*** loss cold bend test is used as a quality control check to ensure the bar's formability and the ***existence of ductility***. This test typically requires the sample to be bent around a forming pin to 180° angles, and visually inspected for development of any surface cracks. International standards specify requirements for the radius of the forming pin R, as a function of the bar diameter d_o as follows;

$$R = d_o \quad d_o < 25mm,$$
$$R = 1.5 * d_o \quad d_o < 25mm,$$

In addition, most rebar standards require that the bend test be completed in one continuous test stroke.

Cold Bent Test





Mode of Failure

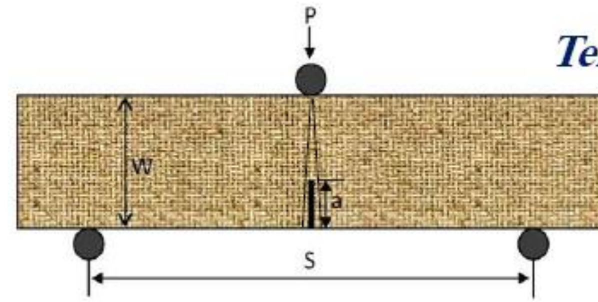
Tension
Lack of ductility

Compression

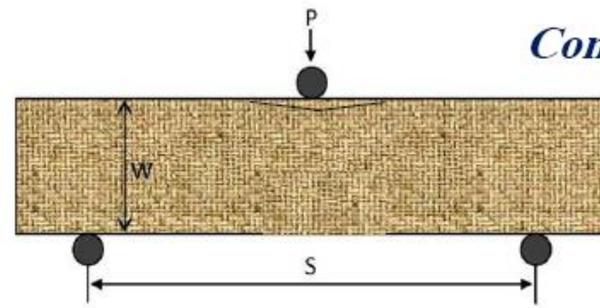
Shear
Small pin radius

Cold Bent Test

Tension



Compression



Shear